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The onset of Taylor-Görtler vortices in the time-dependent Couette flow induced by an impulsively imposed shear stress

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Abstract−The onset of Taylor-Görtler instability induced by an impulsively started rotating cylinder with constant shear stress was analyzed by using propagation theory based on linear theory and momentary instability concept. It is well-known that the primary transient Couette flow is laminar but secondary motion sets in when the inner cylinder velocity exceeds a certain critical value. The dimensionless critical time τ_c to mark the onset of instability is presented here as a function of the modified Taylor number T. For the deep-pool case of small τ, since the inner cylinder velocity increases as $V_i \propto \sqrt{t}$ in the present impulsive shear system, the present system is more stable than impulsive started case (V_i=constant). Based on the present τ_c and the Foster's [1969] comment, the manifest stability guideline is suggested.

Key words: Taylor-Görtler Vortex, Time-Dependent Couette Flow, Constant Shear, Propagation Theory

INTRODUCTION

The stability of time-dependent Couette flow has been extensively studied theoretically and experimentally. The related flow driven by centrifugal forces occurs in a wide range of scientific and engineering fields, such as polymer processing and Bridgman crystal growth systems [Schweizer and Scriven, 1983; Yeckel and Derby, 2000]. The onset of secondary motion when an inner cylinder is impulsively accelerated with constant angular velocity was investigated experimentally by Chen and Christensen [1967], Kirchner and Chen [1970], Liu [1971] and Kasagi and Hirata [1975]. And, a related stability analysis has been conducted by using the amplification theory [Chen and Kirchner, 1971; Kasagi and Hirata, 1975], the frozen-time model [Chen and Kirchner, 1971], maximum-Taylor-number criterion [Tan and Thorpe, 2003] and propagation theory [Kim et al., 2004; Kim and Choi, 2004, 2005]. Among these, the amplification theory and the frozen-time model are quite popular. The amplification theory is the initial value approach where the randomly-selected initial disturbances are introduced into the linearized transient Navier-Stokes equation, and the growth of these disturbances is tracked by integration of the initial value problem. The onset time of secondary motion is defined as the time when the disturbance kinetic energy grows 10^3-fold its initial value. In the frozen-time model, the basic velocity field is frozen at a time t_c, and the critical Taylor number is obtained by solving the linearized Navier-Stokes equation which is frozen at a given time.

Recently, Tan and Thorpe [2003] suggested a simple instability analysis, so-called the maximum-Taylor-number criterion. In this model, a newly defined transient-Taylor number is introduced and the critical conditions are determined by letting the maximum value of this transient-Taylor number to the well-known steady state critical Rayleigh number by considering the similarity between the time-dependent Bénard-Rayleigh problem and the time-dependent Taylor problem. Since it is well-known that the governing equation and the boundary conditions for these two problems are very similar and even approximately identical [Chandrasekhar, 1961], the above-mentioned methods which were originally devised for the Bénard-Rayleigh problem have been extended into the Taylor problem by taking advantage of the similarities.

Another model to analyze the time-dependent convective instability problem is the propagation theory [Kim et al., 2002, 2004, 2005]. This theory has dealt with the onset of thermal instabilities in the initially motionless fluid layers heated rapidly from below, which assumes that at $t=t_c$ infinitesimal temperature disturbances are propagated mainly within the thermal penetration depth Δ_T. In this method the length scales in disturbance variables and the stability parameters are rescaled with thermal penetration depth Δ_T. In a usual deep-pool conduction system of $\Delta_T \propto \sqrt{\alpha t}$, the most important parameter becomes the time-dependent Rayleigh number, which is yielded by replacing the length scale in the Rayleigh number which is yielded by replacing the length scale in the Rayleigh number with Δ_T. Here α is the thermal diffusivity. The resulting stability criteria have compared well with experimental data of various systems such as solidification [Hwang and Choi, 1996], Marangoni-Bénard convection [Kang and Choi, 1997; Kang et al., 2000] and Rayleigh-Bénard convection in porous media [Yoon and Choi, 1989].

Here we will try to analyze the time-dependent Taylor problem in the flow induced by an impulsive shear by employing the propagation theory and the frozen-time model. In this case, the rotation of an inner cylinder is driven by an imposed constant wall shear stress, and the inner cylinder rotates with angular velocity proportional to the square root of time for a certain time. This system corresponding to the Rayleigh-Bénard system heated from below with constant heat flux. The resulting theoretical results will be compared with each other, and the effect of rotating history on the onset of vortices will be studied.

STABILITY ANALYSIS

1. Governing Equations

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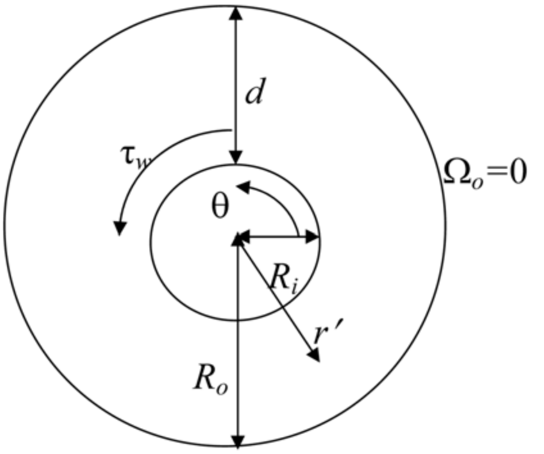

Fig. 1. Schematic diagram of system considered here.

The system considered here is a Newtonian fluid confined between the two concentric cylinders of radii R_i and $R_o(>R_i)$. Let the axis of inner cylinder be along the z′ axis of a cylindrical coordinate system (r′, θ, z′). At the time t=0, the inner cylinder starts to rotate by an imposed constant shear stress τ_w and outer cylinder is kept stationary. A schematic diagram of the basic system is shown in Fig. 1. The governing equations of the present flow field is expressed by

$$\nabla\cdot\mathbf{U}=0, \tag{1}$$

$$\left\{\frac{\partial}{\partial t}+\mathbf{U}\cdot\nabla\right\}\mathbf{U}=-\frac{1}{\rho}\nabla P+\nu\nabla^2\mathbf{U}, \tag{2}$$

where **U**, P, ν and ρ represent the velocity vector, the dynamic pressure, the kinematic viscosity and the density, respectively.

The important parameters to describe the present system are the Taylor number T, the Reynolds number Re and the radius ratio η defined as

$$T=\frac{V_r^2d^2}{\nu^2}\frac{d}{R_i},\quad Ta=\frac{V_i^2d^2}{\nu^2}\frac{d}{R_i},\quad Re=\frac{V_rd}{\nu}\quad\text{and}\quad \eta=\frac{R_o}{R_i},$$

where $V_r=\tau_w d/(\rho\nu)$ and $d=(R_o-R_i)$. Here V_i is the inner cylinder velocity and has the relation of $V_r\sqrt{\tau}$ for the deep-pool case of small τ. In case of a very slow rotating speed the basic velocity profile finally becomes time-independent and Taylor vortices appear at $T=T_c$.

But for an impulsively started system of large T, the secondary motion onsets before the basic flow field becomes fully-developed and time-independent, and therefore in this transient period the critical time to mark the onset of Taylor-Görtler vortices is important problem. For the present system, the basic velocity field for developing Couette flow is represented for the case of constant physical properties in dimensionless form:

$$\frac{\partial v_0}{\partial v}=DD_*v_0, \tag{3}$$

with the following initial and boundary conditions,

$$v_0(0,r)=0,\quad \frac{dv_0}{dr}\left(\tau,\frac{\eta}{1-\eta}\right)=-1\quad\text{and}\quad v_0\left(\tau,\frac{\eta}{1-\eta}\right)=0, \tag{4}$$

where $\tau=\nu t/d^2$, $v_0=V_0/V_r$, $D=\partial/\partial r$, $D_*=\partial/\partial r+1/r$ and $r=r'/d$.

The exact solutions of Eqs. (3) and (4) can be obtained by using the Laplace transform as [Carslaw and Jaeger, 1959];

$$v_0(\tau,r)=\frac{\eta}{1-\eta}\ln\frac{1}{r(1-\eta)}+\frac{\eta}{1-\eta}\sum_{n=0}^{\infty}Q(\lambda_n,\eta)[J_0(\lambda_nr)Y_1(\lambda_n)$$
$$-J_1(\lambda_n)Y_0(\lambda_nr)]\exp\left\{-\lambda_n^2\tau\left(\frac{1-\eta}{\eta}\right)^2\right\} \tag{5a}$$

where J and Y are Bessel functions and $\xi=r(1-\eta)/\eta$. The function Q is

$$Q(\lambda_n,\eta)=\frac{\pi}{\lambda_n\{[J_1(\lambda_n)/J_0(\lambda_n/\eta)]^2-1\}}. \tag{5b}$$

The λ_n's are the roots of the equation

$$J_1(\lambda_n)Y_0\left(\lambda_n\frac{1}{\eta}\right)-Y_1(\lambda_n)J_0\left(\lambda_n\frac{1}{\eta}\right)=0. \tag{5c}$$

For the limiting case of small τ the above solution can be reduced as

$$v_0(\tau,\zeta)=2\left(\frac{1}{\sqrt{\tau}}+\frac{1-\eta}{\eta}\zeta\right)^{-1}$$
$$\left[\text{ierfc}\left(\frac{\zeta}{2}\right)-\frac{\{4+\zeta\sqrt{\tau}(\eta/1-\eta)^{-1}\}}{4\{\zeta+\eta/(1-\eta)(1/\sqrt{\tau})\}}\text{i}^2\text{erfc}\left(\frac{\zeta}{2}\right)+\cdots\right], \tag{6}$$

where $\zeta=y/\sqrt{\tau}$ and $y=(r'-R_i)/d$. The present study concerns the case of $\eta\rightarrow1$, i.e., narrow-gap approximation. For this limiting case, the gap size $d(=R_o-R_i)$ is small compared to mean radius $(R_o+R_i)/2$ and there is no need to distinguish between $D_*(=\partial/\partial r+1/r)$ and $D(=\partial/\partial r)$ under this approximation [Chandrasekhar, 1961], i.e., the effect of curvature can be neglected. Since the above solution does not work so well for $\eta\rightarrow1$ case, the above basic flow solutions can be reduced as

$$\bar{v}_0(\tau,y)=1-y-2\sum_{n=1}^{\infty}\frac{1}{\mu_n^2}\cos(\mu_ny)\exp(-\mu_n^2\tau), \tag{7a}$$

$$v_0(\tau,\zeta)=\sqrt{4\tau}\sum_{n=0}^{\infty}(-1)^n\left\{\text{ierfc}\left(\frac{n}{\sqrt{\tau}}+\frac{\zeta}{2}\right)-\text{ierfc}\left(\frac{n+1}{\sqrt{\tau}}-\frac{\zeta}{2}\right)\right\}, \tag{7b}$$

where $\bar{v}_0(\tau, y)$, $v_0(\tau, \zeta)$, $\mu_n=(n-1/2)\pi$, $\text{ierfc}(\chi)=1/\sqrt{\pi}\exp(-\chi^2)-\chi\text{erfc}(\chi)$ and erfc denotes the complementary error function. Furthermore, for the deep-pool systems of small τ, where the boundary-layer thickness is much smaller than gap size, the above solution can be approximated as

$$v_0=\sqrt{4\tau}\,\text{ierfc}\left(\frac{\zeta}{2}\right) \tag{8}$$

and this approximation is adopted for the limiting case of $\tau\rightarrow0$. A comparison between the exact solution of Eq. (5) and the approximate solution of Eq. (8) is given in Fig. 2. As shown in this figure, Eq. (8) is a good approximation of Eq. (5) for $\tau<0.05$.

The following relation can be obtained by considering the chain rule for partial derivatives,

$$\frac{\partial\bar{v}_0^*}{\partial\tau}=\frac{\partial v_0^*}{\partial\tau}+\frac{\partial v_0^*}{\partial\zeta}\frac{\partial\zeta}{\partial\tau}=\frac{\partial v_0^*}{\partial\tau}-\frac{\zeta}{2\tau}\frac{\partial v_0^*}{\partial\zeta}, \tag{9}$$

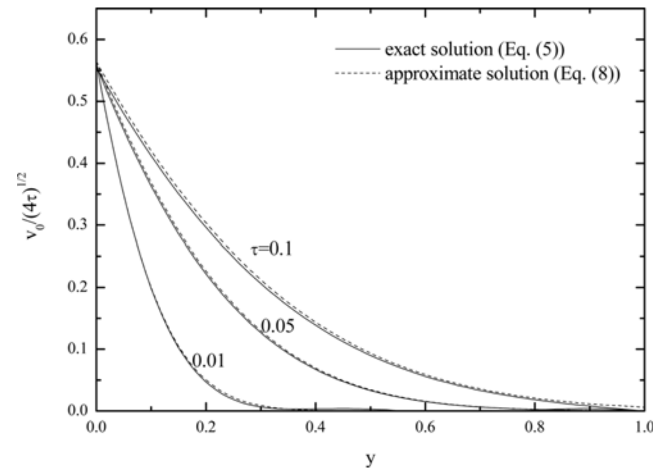

Fig. 2. Base flow profiles for the case of η=0.9.

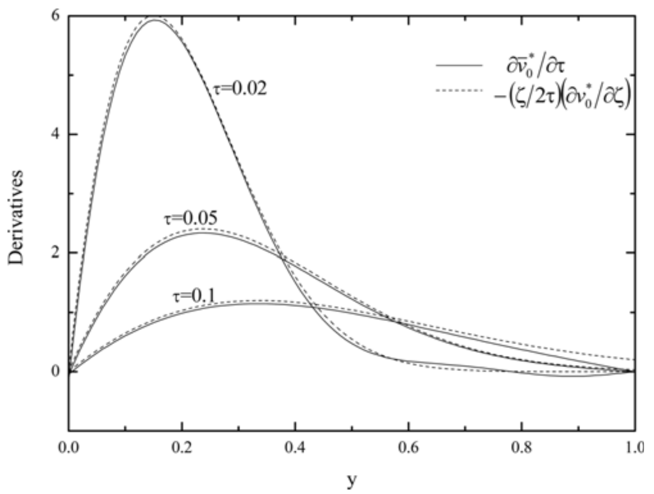

Fig. 3. Comparison of the time derivatives of the primary velocity.

where $\bar{v}_0^*=\bar{v}_0/\sqrt{\tau}$ and $v_0^*=v_0/\sqrt{\tau}$. The first term in the right column is negligible with respect to the second term, as shown in Fig. 3. Therefore, $\partial\bar{v}_0^*/\partial\tau=-(\zeta/2\tau)(v_0^*/\partial\zeta)$ is valid for the limiting case of $\tau\to 0$, and this relation implies that the base flow has self-similar characteristics.

2. Stability Equation

The typical disturbances which are observed experimentally are the axisymmetric ones having the following forms [Chandrasekhar, 1961]:

$$(U_1, V_1, P_1)=(u', v', p')\cos kz' \quad (10a)$$

$$W_1=w'\sin kz' \quad (10b)$$

where k is the wave number and the primed quantities representing disturbance amplitudes are a function of r′ and t. Under linear theory the stability equations of amplitude functions are obtained when w′ and p′ are eliminated. Under the narrow-gap approximation, where $\partial/\partial r'+1/r'\approx\partial/\partial r'$, the resulting dimensionless disturbance equations are represented by

$$\left(\frac{\partial^2}{\partial y^2}-a^2-\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2}{\partial y^2}-a^2\right)u=v_0a^2v \quad (11)$$

$$\left(\frac{\partial^2}{\partial y^2}-a^2-\frac{\partial}{\partial\tau}\right)v=2T\left(\frac{\partial v_0}{\partial y}\right)a^2u \quad (12)$$

with proper boundary conditions,

$$u=\frac{\partial u}{\partial y}=\frac{\partial v}{\partial y}=0 \text{ at } y=0, \quad (13a)$$

$$u=\frac{\partial u}{\partial y}=v=0 \text{ at } y=1, \quad (13b)$$

where $u=d^2u'/(\nu R_i)$, $v=2u'\rho\nu/(\tau_w d)$ and a=kd. The subscript '0' denotes the basic state and a represents the dimensionless vertical wave number. It should be noted that the radial velocity component u′ is nondimensionalized by $\nu R_i/d^2$ rather than V_r.

3. Propagation Theory

The propagation theory employed to find the onset time of secondary motion, *i.e.*, the critical time t_c is based on the assumption that in deep-pool systems of small time the perturbed angular velocity component V_1 is propagated mainly within the hydrodynamic boundary-layer thickness $\Delta\propto\sqrt{\nu t}$ at the onset time of secondary flow, and the following scale relations are valid for perturbed quantities from the linearized equations of Eq. (2):

$$\nu\frac{\partial^2 u'}{\partial r'^2}\sim\nu\frac{u'}{\Delta^2}\sim\frac{V_0}{r'}v'\sim\frac{V_0}{R_i}v', \quad (14)$$

$$u'\frac{\partial V_0}{\partial r'}\sim u'\frac{V_0}{\Delta}\sim\nu\frac{\partial^2 v'}{\partial r'^2}\sim\nu\frac{v'}{\Delta^2}, \quad (15)$$

from the balance between viscous, inertia and centrifugal terms in Eq. (2). Now, based on the relation (14), the following amplitude relation is obtained in dimensionless form:

$$\frac{1}{v_0}\frac{u}{v}\sim\frac{\Delta^2}{d^2}(=\delta^2)\sim\tau \quad (16)$$

where $v_0\sim\sqrt{\tau}$ and $\delta(\propto\sqrt{\tau})$ is the usual dimensionless boundary-layer thickness. The relation (15) yields:

$$\frac{\partial V_0}{\partial r'}\sim\frac{\nu R_i}{\Delta^4V_0}=\frac{V_i}{\Delta}\left(\frac{\Delta^3V_i^2}{\nu R_i}\right)^{-1}=\frac{V_i}{\Delta}Ta_\Delta^{-1} \quad (17)$$

where Ta_Δ is the Taylor number based on the boundary layer thickness Δ. With increasing T both the onset time t_c and corresponding $\Delta(\sim\sqrt{\nu t})$ become smaller and the characteristic value of $T\tau(\Delta/d)^3$ *i.e.* $T\tau^{5/2}$ will become a constant.

There are many possible forms of dimensionless amplitude functions of disturbances like

$$[u(\tau,y), v(\tau,y)]=\lfloor\tau^{n+1/2}\bar{u}^*(\tau,y), \tau^n\bar{v}^*(\tau,y)\rfloor \quad (18)$$

which satisfy the relation (19). Now, we set $\bar{u}^*(\tau, y)=u^*(\tau, \zeta)$ and $\bar{v}^*(\tau, y)=v^*(\tau, \zeta)$ as in Eqs. (5) and (6). If disturbance amplitudes follow the property of the primary flow shown in the relation (8), it is probable that and also $\partial\bar{u}^*/\partial\tau\approx-(\zeta/(2\tau))(\partial u^*/\partial\zeta)$ and $\partial\bar{v}^*/\partial\tau\approx-(\zeta/(2\tau))(\partial v^*/\partial\zeta)$. At this stage the criterion to determine n is necessary. Shen [1961] suggested a momentary instability condition: the temporal growth rate of the kinetic energy of the perturbation velocity should exceed that of the basic velocity at the onset condition of secondary motion. In the present system the dimensionless kinetic energy is defined as

$$E(t)=\frac{1}{2}\|\mathbf{U}^2\| \tag{19}$$

where $\|\cdot\|$ denotes the norm. Since there is no basic flow in θ and z' directions and the condition of $|u/v|\rightarrow 0$ is valid for $\tau\rightarrow 0$ (see the relation (16)), the dimensionless kinetic energy can be divided into the basic one and the perturbed one:

$$E_0(\tau)=\frac{1}{2}\|v_0^2\| \quad \text{and} \quad E_1(\tau)=\frac{1}{2}\|v^2\| \tag{20}$$

Then the temporal growth rates of the basic kinetic energy r_0 and the perturbation energy r_1 are obtained as

$$r_0(\tau)=\frac{1}{E_0}\frac{dE_0}{d\tau} \quad \text{and} \quad r_1(\tau)=\frac{1}{E_1}\frac{dE_1}{d\tau} \tag{21}$$

For the case of n=1/2 the condition of $r_0=r_1$ is fulfilled, which will be discussed later. This condition of $r_0=r_1$ is suggested as a marginal condition like that in Choi et al.'s [2004] Rayleigh-Bénard problem. By the above scaling reasoning we set $u=\tau^2u^*(\zeta)$ and $v=\tau^{1/2}v^*(\zeta)$. For a deep-pool system of small τ, the dimensionless time τ is related with the time for development of the boundary-layer thickness, which plays dual roles of time and length. By using relations (19) and (20) the following self-similar stability equations are obtained with $\partial/\partial\tau=(-\zeta/2\tau)D$ and $\partial^2/\partial y^2=(1/\tau)D^2$ from Eqs. (11) and (12),

$$\left[(D^2-a^{*2})^2+\frac{1}{2}(\zeta D^3-3D^2-a^{*2}\zeta D+2a^{*2})\right]u^*=v_0^*a^{*2}v^*, \tag{22}$$

$$\left(D^2-a^{*2}+\frac{1}{2}\zeta D-\frac{1}{2}\right)v^*=2T^*Dv_0^*u^*, \tag{23}$$

where $D=d/d\zeta$, $a^*=a\sqrt{\tau}$ and $v_0^*=v_0/\sqrt{\tau}$. The proper boundary conditions of no-slip are

$$u^*=Du^*=Dv^*=0 \text{ at } \zeta=0, \tag{24a}$$
$$u^*=Du^*=v^*=0 \text{ as } \zeta\rightarrow\infty. \tag{24b}$$

For a given τ, T^* and a^* are treated as eigenvalues and the minimum value of T^* is found in the plot of T^* vs. a^* under the principle of exchange of stabilities. This produces the earliest time t_c and its corresponding wave number a_c.

The conventional frozen-time model neglects the terms involving $\partial/\partial\tau$ in Eqs. (14) and (15) in amplitude coordinates τ and y. This results in $(D^2-a^{*2})^2u^*=v_0^*a^{*2}v^*$ and $(D^2-a^{*2})v^*=2T^*Dv_0^*u^*$ instead of Eqs. (25) and (26). The minimum T-value is obtained for a given τ_c.

SOLUTION METHOD

In the present study the stability Eqs. (22)-(24) are solved by employing the outward shooting scheme. In order to integrate the stability equations the proper values of D^2v^*, D^3v^* and u^* at $\zeta=0$ are assumed for a given a^*. Since the stability equations and the boundary conditions are all homogeneous, the value of $D^2v^*(0)$ can be assigned arbitrarily and the T^*-value is assumed. This procedure can be understood easily by taking into account characteristics of the eigenvalue problem. After all the values at $\zeta=0$ are provided, this eigenvalue problem can be treated numerically.

Integration is performed from the heated surface $\zeta=0$ to a fictitious outer boundary with the fourth order Runge-Kutta-Gill method. If guessed values of T^*, $D^3v^*(0)$ and $u^*(0)$ are correct, v^*, Dv^* and u^* will vanish at the outer boundary. To improve the initial guesses the Newton-Raphson iteration is used. When convergence is achieved, the outer boundary is increased by a predetermined value and the above procedure is repeated. Since the disturbances decay exponentially outside the boundary-layer thickness, the incremental change of T^* also decays fast with an increase in outer boundary depth. This behavior enables us to extrapolate the eigenvalue T^* to the infinite depth by using the Shank transform. The results of this procedure are presented in Fig. 2, as a plot of T^* vs. a^*. The minimum value of T^*, i.e., $T_c^*=128.08$ at $a_c^*=0.96$, will mark the onset of vortices. In the case of the frozen-time model a similar technique is employed and the characteristic values obtained.

RESULTS AND DISCUSSION

For a single-mode instability the onset time to mark secondary motion is predicted by propagation theory. Based on the results of Fig. 2, the critical conditions to mark the onset of secondary motion are given by

$$\tau_c=6.97T^{-2/5} \text{ and } a_c=0.36T^{1/5} \quad \tau_c<0.01, \tag{25}$$

The above stability condition can be rewritten as a function of Ta:

$$\tau_c=25.41Ta^{-2/3} \text{ and } a_c=0.19Ta^{1/3} \text{ for } \tau_c<0.01, \tag{26}$$

since $Ta=T\tau$. This equation is less convenient in predicting τ_c but it is more useful for comparison with impulsively started (V_i=constant) system where the critical conditions are given as [Kim et al., 2004]:

$$\tau_c=20.05Ta^{-2/3} \text{ and } a_c=0.19Ta^{1/3} \text{ for } \tau_c<0.01. \tag{27}$$

For the case of small τ, since the inner cylinder velocity increases as $V_i\propto\sqrt{t}$ in the present impulsive shear case, this system is more stable than impulsive started case (V_i=constant), even if both cases are unstable to small amplitude disturbances. The resulting normalized amplitude functions of u^* and v^* are shown in Fig. 3. It is shown that v' is propagated mainly within the basic boundary-layer thickness. For a given T, the fastest growing mode of infinitesimal disturbances would set in at $\tau=\tau_c$ with $a=a_c$. The above equations show that τ_c decreases with an increase in T. From distributions of the basic flow (Eq. (8)) and the perturbation quantities, we can obtain the following relation:

$$r_0=r_1=\frac{3}{2\tau_c} \text{ at } \tau=\tau_c \tag{28}$$

The above equation indicates that the propagation theory satisfies Choi et al.'s [2004] criterion on instability of $r_1\geq r_0$. In a frozen-time model, the growth rates of the disturbances are assumed to be zero and the critical conditions are determined under the assumptions of $r_1=0$ and $r_1>>r_0$. These assumptions are contradictory to each other, since $r_0>0$ as shown in Eq. (28). So, the frozen-time model seems to have lost its theoretical background.

Now, the domain of time is extended to $\tau>0.05$ by keeping Eqs. (22) and (23) and using Eq. (11). In Eq. (27) the infinite upper boundary is replaced with the finite one y=1, *i.e.*, $\zeta=1/\sqrt{\tau_c}$ and in Eq. (25) and (26) T^* and a^* are replaced with $\tau_c^{5/2}T$ and $\tau_c^{1/2}a$. Also, in Eq. (11) τ is fixed as τ_c but ζ is maintained. Since τ is the fixed parameter,

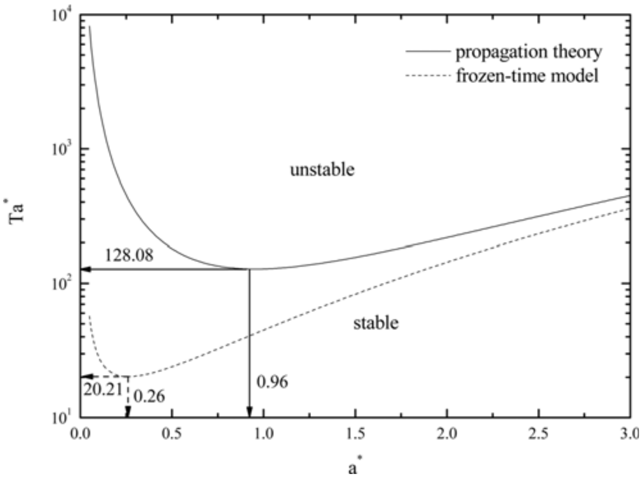

Fig. 4. Marginal stability curve under the principle of exchange of stabilities for small time of $\tau_c \to 0$ from the propagation theory.

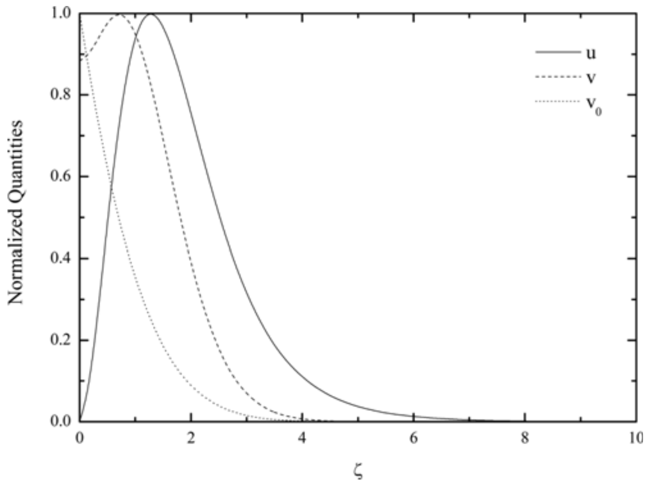

Fig. 5. Amplitude profiles at $\tau = \tau_c$ for small time of $\tau_c \to 0$ from the propagation theory.

the resulting stability equations are a function of ζ only and the spirit of relations (19) and (20) is still alive. For a given τ_c the minimum T-value and its corresponding wave number a_c are obtained. The results are summarized in Fig. 4, wherein those obtained from the conventional frozen-time model are also shown. For $\tau<0.01$ the predictions from the propagation theory are the same as those in deep-pool systems (Eq. (28)). For large τ_c they approach the critical value of $T_c=1200$ and $a_c=2.52$ since the basic flow velocity profile is linear in the steady state. This critical value of $T_c=1200$ is smaller than that of $Ta_c=1695$ for the constant velocity case, and this trend is similar to the Rayleigh-Bénard problem where $Ra_{q,c}=1296$ is smaller than $Ra_c=1708$. Here Ra_q is the Rayleigh number based on the heat flux and corresponds to the present Taylor number based on the wall shear stress. It is known that for small τ the frozen-time model yields the lower bounds of τ_c and the terms involving $\partial/\partial\tau$ in Eqs. (25) and (26) stabilize the system.

Recently Tan and Thorpe [2003] suggested a simple instability analysis. They defined transient Taylor number Ta_t as

$$Ta_t = \frac{d^3 y^5 (dV_0/dy)^2}{\nu^2 R_i}, \tag{29}$$

and assumed that at the detection time of manifest convection the following relation is maintained, based on Eq. (8):

$$\text{Maximum of}(Ta_t)=817, \tag{30}$$

which is satisfied by $y_{max}=1.9349\sqrt{\tau}$. This results in $T^*=175.91$. This relation corresponds to the system of $\eta \to 1$. In the R-B system heated from below with constant heat flux, the value 817 corresponds to the critical Rayleigh number for the rigid-free boundaries. Tan and Thorpe's [2003] predictions agree with the present τ_c-values. It is interesting that common relation is involved in the above results: T^*=constant.

Foster [1969] commented that with correct dimensional relations the relation of $\tau_m \approx 4\tau_c$ would be kept for a large Rayleigh-number R-B problem. This means that a fastest growing mode of instabilities, which set in at $\tau=\tau_c$, will grow with time until manifest convection is detected near the whole bottom boundary at $\tau=\tau_m$. For the impulsively started system, Chen and Kirchner [1971] and Kim and Choi [2005] reported a similar trend based on the amplification the-

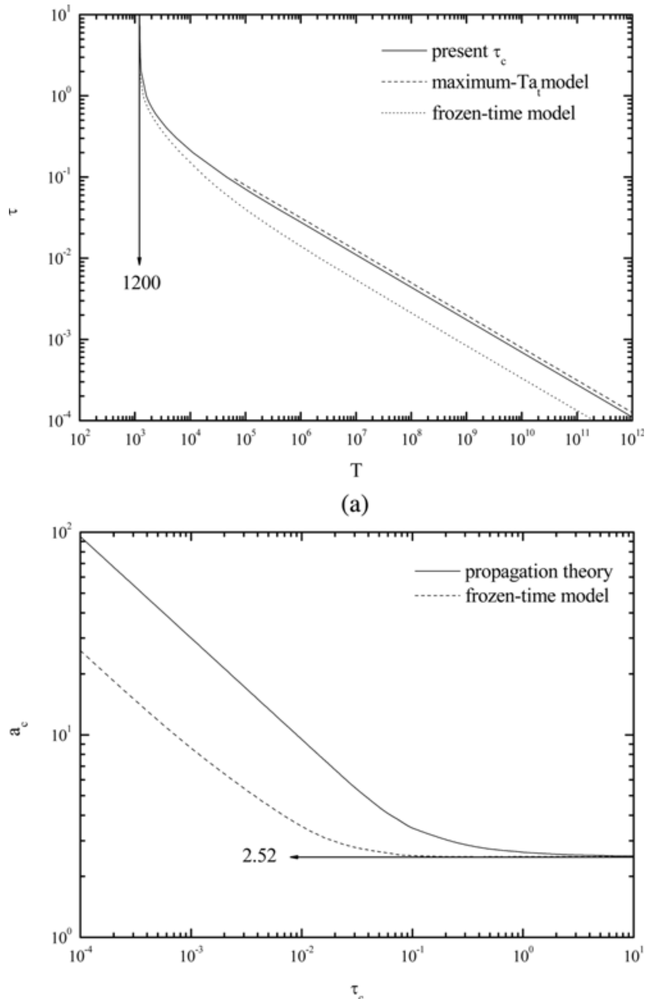

Fig. 6. Stability conditions: (a) critical time and (b) critical wave number.

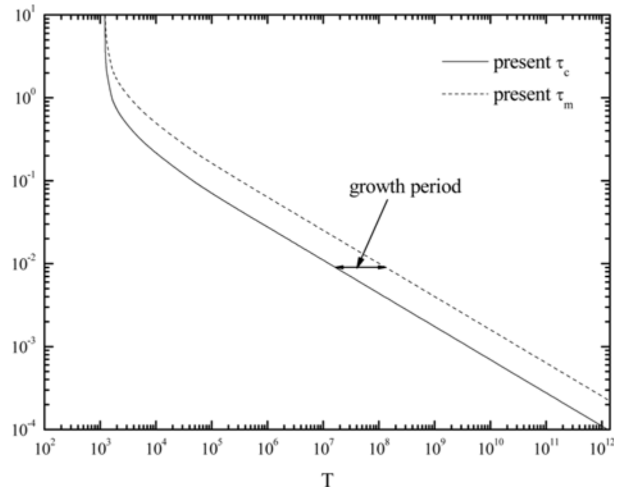

Fig. 7. Suggested detection time of instability and growth period.

ory and the propagation theory, respectively. From the Foster's comment ($\tau_o \cong 4\tau_c$), the following relation can be obtained:

$$\tau_m = 4\tau_c = 101.64\mathrm{Ta}^{-2/3} \text{ for } \tau_c < 0.01. \quad (31)$$

based on the results of Eq. (26). Since the inner cylinder velocity increases continuously during the growth period ($\tau_c \le \tau \le \tau_m$), the detection time τ_m is obtained from Eq. (31) by replacing Ta with $\mathrm{T}\tau_m$:

$$\tau_m = 16.00\mathrm{T}^{-2/5} \text{ for } \tau_m \le 0.01. \quad (32)$$

This procedure is applied in transient diffusive systems [Kim et al., 2005], successfully. A similar trend is expected for $\tau_c > 0.01$. It seems evident that convective motion is very weak during $t_c \le t \le t_m$ since the related momentum transport is well represented by the diffusion state. This growth period is described in Fig. 7.

CONCLUSIONS

The onset of secondary motion in the flow by a suddenly started rotating cylinder with constant shear has been analyzed by using linear stability theory. Propagation theory has been employed to predict the critical time to mark the onset of convective instability. Also, the stability guidelines and the critical Taylor number T_c have been suggested. The present impulsive shear system is more stable than the impulsive started system, even if both systems are unstable when the inner cylinder velocity exceeds a certain value. It seems that the propagation theory is a powerful method to predict the stability criteria reasonably well in the simple systems, hydrodynamic or thermal, of which the basic states involve transient diffusion processes.

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